An Analysis of the Large Deflection of a Cantilever Beam by the Moiré Method*

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An analysis of the elastic deflection of a beam can be performed by the moiré method. The authors extended to the analysis of the large deflection of a cantilever beam. In general, the fringe patterns formed in a cantilever beam to which a large deflection is given are curved due to the effect of strain. The authors found that these curved fringes may be transformed into nearly straight ones by introducing a proper misalignment into the master grid.

In this paper, a new theory for analyzing correctly the large deflecton of a cantilever beam is proposed, and the accuracy in measurement by using this theory is discussed.

1. Introduction

The moiré method or the moiré topographic method using two sheets of parallel line grids is an effective means for measuring the strain or the deflection at any point in a complicated object. Some problems, for example, two dimensional strain analysis,[(1)] thermal stress analysis[(2)] and the deflection analysis of a bent plate[(3)] were performed by using these methods. As an another example, an analysis of the elastic deflection of a beam[(4)] was carried out.

The object of this study is to analyze correctly the large deflection of a cantilever beam with a single row of slotted holes.

In general, the deflection analysis of a beam by the moirē method is performed as follows. A model grid is pasted or printed photographically on the beam in such a way that the model grid lines are ditrected along the longitudinal axis of the beam. When a large deflection is given to the beam, the model grid lines are rotated and displaced from the initial positions. When a master grid is superposed upon the deformed model grid, the moiré fringe patterns formed are curved due to the effect of strain. Therefore, in the case of a large deflection, it is necessary to eliminate the effect of strain from fringe patterns in order to obtain correctly the deflection. For this purpose, it is described that the introduction of a proper misalignment into the master grid is effective. And a theory of the deflection analysis by using this procedure is proposed. Further, the accuracy in measurement by uing this theory is discussed.

2. The theory for measuring a large deflection

In this analysis, orthogonal coordinates (x, y) will be used. Let the lougitudinal and transverse axes of a beam be taken as the x and y axes, respectively, and a model grid with an equispaced pitch p_o be pasted on the beam in such a way that the model grid lines are directed along the longitudinal axis of the beam. The equation for these grid lines is expressed as

$$y = l p_o (l = 0, \pm 1, \pm 2, \cdots\cdots), \tag{1}$$

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where l is a grid line index. When a lateral load is applied to the beam, the deformed model grid lines contain both informations of deflection and strain. But, when the deflecton is measured in the neighborhood of the neutral axis of the beam, the equation for the deformed model grid lines can be written as

$$y=-\tan i\cdot x+\frac{lp_o}{\cos i}, \tag{2}$$

where i is a slope.

On the other hand, the equation for the master grid lines with an amount of misalignment θ is expressed by

$$y=\tan \theta\cdot x+\frac{kp}{\cos \theta}=\tan \theta\cdot x+\frac{kp_o}{(1+\lambda)\cos \theta}(k=0,\ \pm1,\ \pm2,\ \cdots\cdots), \tag{3}$$

where k is a grid line index, p is a pitch and λ is an amount of mismatch.

By superposing the two grids expressed by Eqs. (2) and (3), moiré fringes are formed, which are expressed by the following fringe index equation.

$$k-l=m\ (m=0,\ \pm1,\ \pm2,\cdots\cdots), \tag{4}$$

where m is a fringe index.

Substituting Eqs. (2) and (3) into Eq. (4), we have a fringe-slope equation as follows:

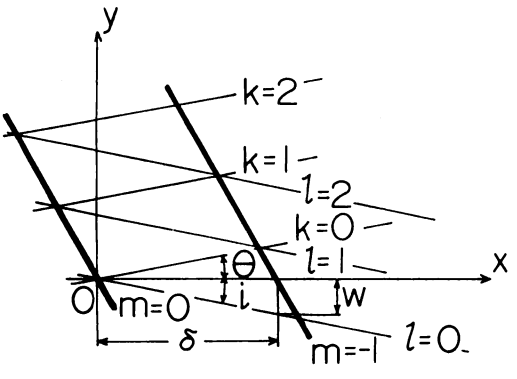


Fig. 1 Geometry of moiré fringe.

$$y=\frac{1}{(1+\lambda)\cos\theta-\cos i}\left\{(1+\lambda)\sin\theta+\sin i\right\}x+mp(1+\lambda). \tag{5}$$

The thick lines shown in Fig. 1 indicate these fringes.

Putting $y=0$ in Eq. (5), we have

$$x=\frac{mp(1+\lambda)}{(1+\lambda)\sin\theta+\sin i}. \tag{6}$$

Now, let the fringe indices of any two adjacent fringes on the x axis be m_n and m_{n+1}, and also the interval between these two adjacent fringes be δ. Then, from Eq. (6) we obtain

$$\delta=(m_{n+1}-m_n)\frac{p(1+\lambda)}{(1+\lambda)\sin\theta+\sin i}=M\frac{p(1+\lambda)}{(1+\lambda)\sin\theta+\sin i}, \tag{7}$$

where M is the difference in fringe indices between two adjacent fringes and has a value of ±1.

When the effect of the strain is taken into consideration in Eq. (2), it is transformed into

$$y=-\tan i\cdot x+\frac{lp_o(1+\varepsilon)}{\cos i}. \tag{2'}$$

Then, the equation for the interfringe-spacing is expressed in place of Eq. (7) by

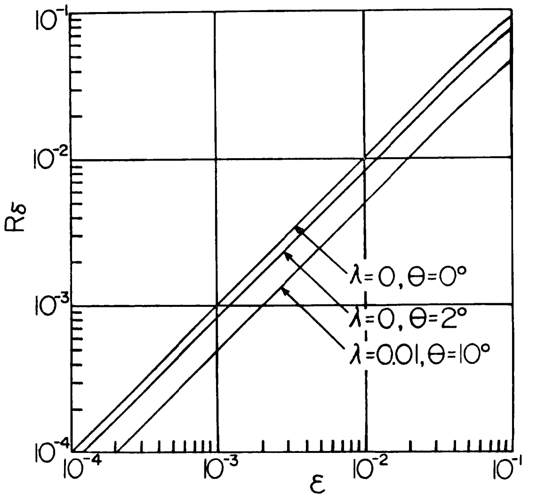


Fig. 2 Relative error due to the tensile strain.

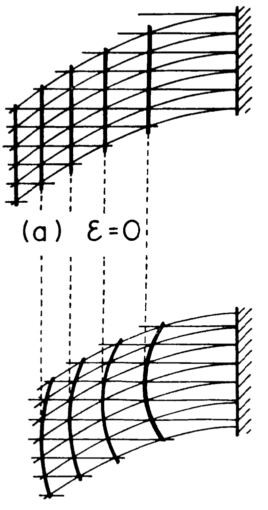


Fig. 3 Model fringe patterns formed under the condition of $\theta=0°$ and $\lambda=0$

$$\delta\varepsilon = M\frac{p(1+\lambda)\,(1+\varepsilon)}{(1+\lambda)(1+\varepsilon)\sin\theta+\sin i}. \tag{8}$$

The relative error $R\delta$ between the interfringe-spacings $\delta\varepsilon$ and δ is given by

$$R\delta = \left|\frac{\delta-\delta\varepsilon}{\delta\varepsilon}\right|. \tag{9}$$

Fig. 2 shows the change in the relative error $R\delta$ due to the tensile strain ε under the condition of $i=10°$ constant. In the figure, θ and λ are used as parameters. As is apparent in the figure, it is seen that the relative error is decreased with the increase of misalignment and mismatch values. Figs. 3(a) and (b) show the model fringe patterns formed in cantilever beams for the cases that $\varepsilon=0$ and $\varepsilon\neq 0$, respectively, under the condition that $\theta=0$ and $\lambda=0$. As is apparent in the figure, the fringes are straight and curved for the cases that $\varepsilon=0$ and $\varepsilon\neq 0$, respectively. As a matter of course, it is easier to measure the interfringe-spacings in Fig. 3(a) than in Fig. 3(b). The introduction of a proper misalignment into the master grid can transform the curved fringes into nearly straight ones. Fig. 4 shows the fringes transformed from those of Fig. 3(b) by giving the misalignment of $\theta=30°$.

This transformation may be performed easily by using a rotating device of the master grid.

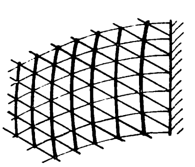


Fig. 4 Model fringe patterns formed under the condition of $\theta=30°$ and $\lambda=0$.

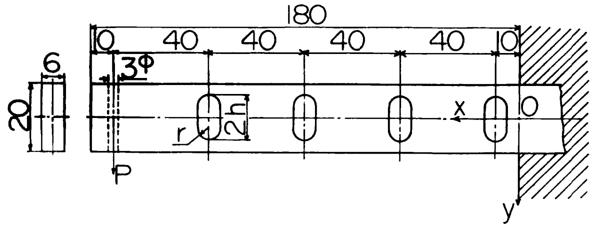


Fig. 5 Shape and dimensions of the specimen.

Now, Eq. (7) yields the expression of slope i as

$$i = \arcsin(1+\lambda)\left(\frac{Mp}{\delta} - \sin\theta\right). \tag{10}$$

The sign of M plays an important role in obtaining correct slopes, but the determination of this sign from one fringe pattern is difficult. However, this sign can easily be determined by using the procedure proposed by the authors previously[5]. When the master grid superposed upon the deformed model grid is shifted in the direction parellol to the y axis by an amount Δ, Eq. (6) is transformed into

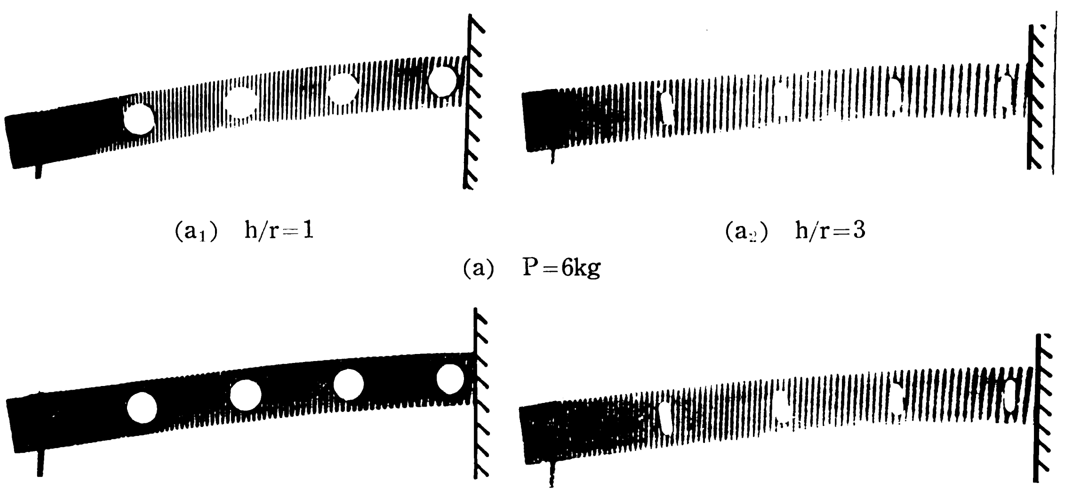
$$x = \frac{mp(1+\lambda)}{(1+\lambda)\sin\theta + \sin i} + \frac{\Delta(1+\lambda)\cos\theta}{(1+\lambda)\sin\theta + \sin i}. \tag{11}$$

The first term in the right hand side of the above equation has a constant value under a given deflection, and the second term shows the displacement of fringes produced by the shift of the master grid by Δ. The second term is useful for determining the sign of M. When the master grid is given a positive shifting Δ (when the shifing is given in the direction toward increasing the positive coordinate, the sigt of Δ is prescribed to be positive), the fringes cutting the x axis are shifted toward the positive or negative direction (the positive or negative direction means that of increasing the absolute value of coordinate on the axis), according as $(1+\lambda)\sin\theta + \sin i$ is larger or smaller than zero, respectively. This direction of the fringe shift can be correlated with the sign of M as follows. If the fringes cutting the x axis are shifted toward the positive or negative direction, then M is 1 or -1, respectively.

Besides, let the difference in the deflection of the beam between any two adjacent fringes be w, then we obtain the relation

$$w = \delta \tan \arcsin\,(1+\lambda)\left(\frac{Mp}{\delta} - \sin\theta\right), \tag{12}$$

since $w = \delta \tan i$ from Eq. (10). Therefore, the total deflection of the beam at the fringes of nth order is obtained by summing up the individual deflection as follows:



(a_1) h/r=1 (a_2) h/r=3

(a) P=6kg

(b_1) h/r=1 (b_2) h/r=3

(b) P=8kg

Fig. 6 Some examples of moiré fringe patterns.

$$W=\sum_{j=1}^{n} w_j.$$

3. Experimental results

In this experiment, the nitro-cellulose resin with a camphor content of 24% was used, which had the elastic modulus of $210 kg/mm^2$ and the Poisson's ratio of 0.37. The shape and dimensions of the specimen adopted are shown in Fig. 5. As is seen in the figure, a cantilever beam with a single row of four slotted holes was used, and the ratio h/r of the slotted holes was varied in four ways, that is, ∞, 3, 1.5 and 1. Four kinds of lateral loads, that is, 2, 4, 6 and 8kg were applied to the beam at the position shown in the figure.

Figs. 6(a) to (d) represent the moiré fringe patterns obtained, in which the grid with the pitch of p=0.495mm were used under the condition that $\theta=6°$ and $\lambda=0$. Figs. 7(a) to (d) and 8(a) to (d) show the distributions of slopes and deflectiones obtained from these fringe patterns, respectively.

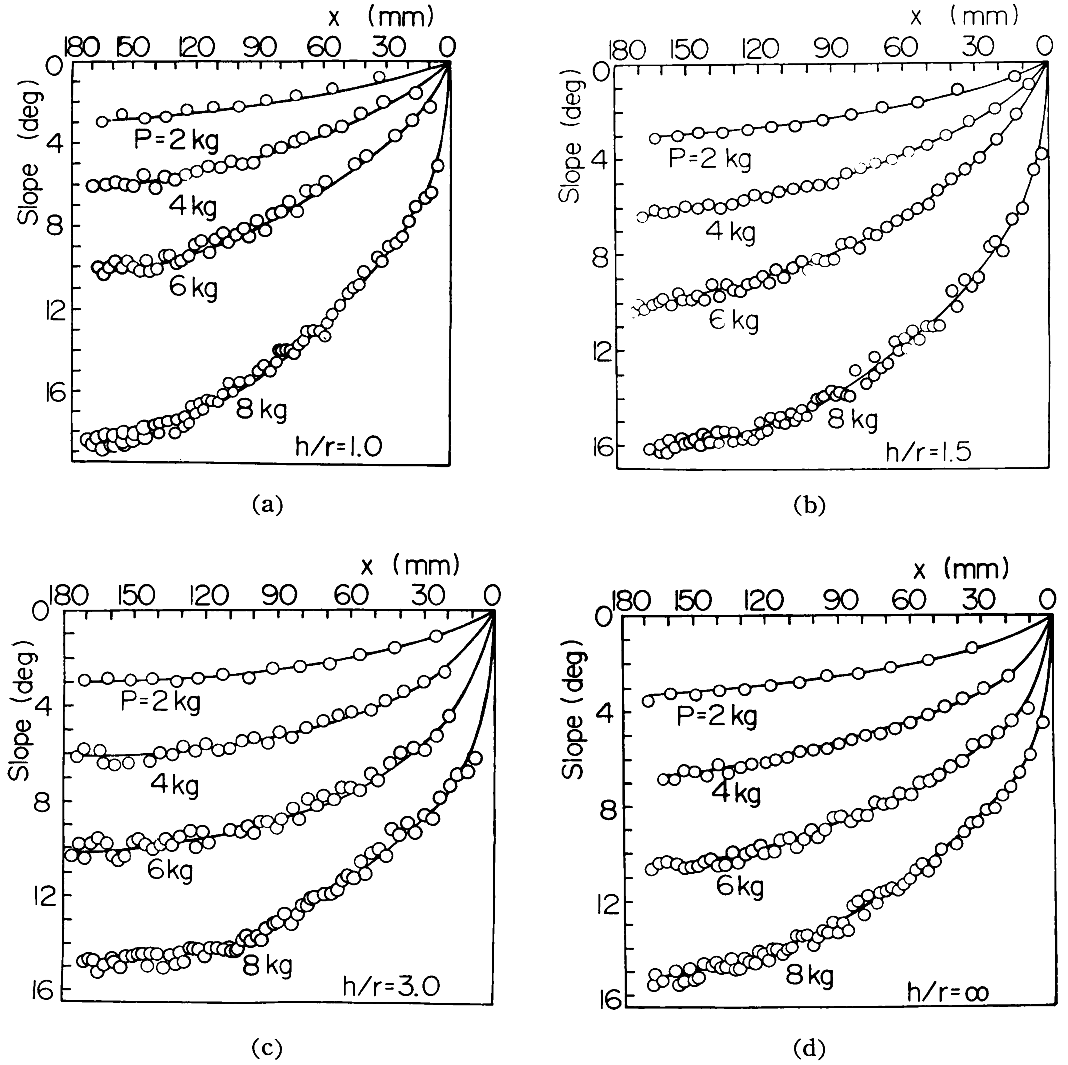
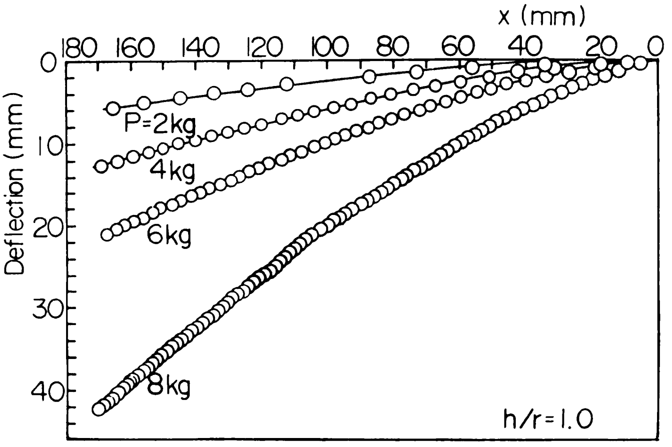
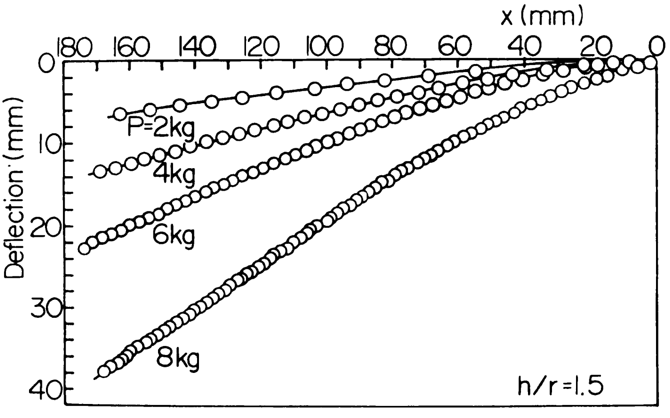


Fig. 7 Distributions of slopes along the longitudinal axis of the beam.

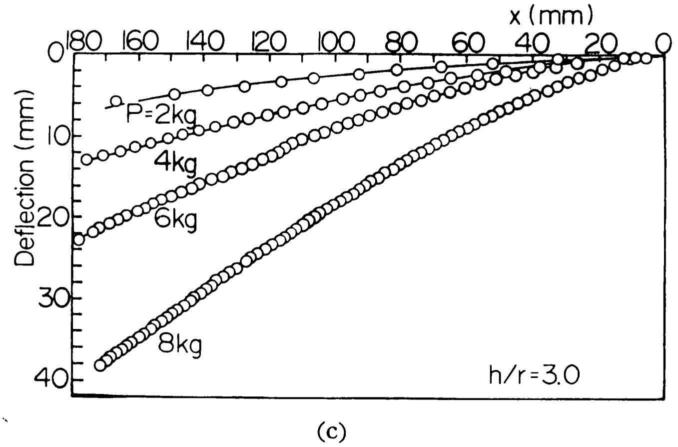
In the figures, full circles show the experimental results, and full lines in Fig. 8 indicate the results obtained through the measurement by a caliper. A good agreement is seen in both expermental results in Fig. 8.



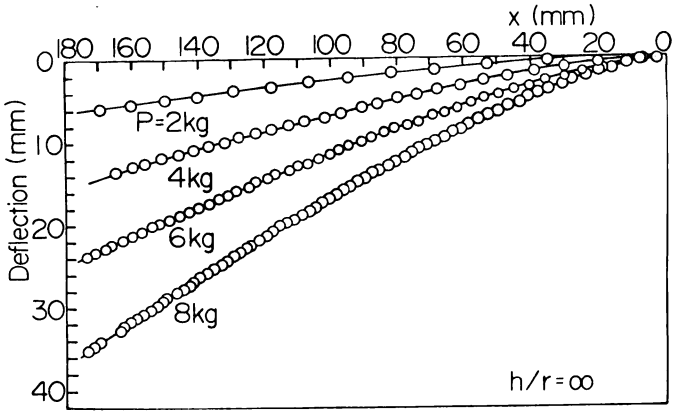
(a)



(b)



(c)



(d)

Fig. 8 Distributions of deflections along the longitudinal axis of the beam.

4. Conclusions

From the experimental results mentoned above, the folloming conclusions may be drawn.

(1) The slope and deflection of a cantilever beam to which a large deflection is given, can be obtained by using Eqs. (10) and (11), respectively.

(2) The curved fringe patterns formed in a cantilever beam to which a large deflection is given, can be transformed into nearly straight ones by introducing a proper misalignment into the master grid, thus raising the accuracy in the measurement of interfringe-spacings.

(3) The sign of M introduced into Eqs. (10) and (12) has an important role for obtaining the correct deflection, and can easily be determined by the master grid shifting procedure proposed by the authors previously.

References

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